# *Relativistic theory of time and frequency transfers*

Pierre Teyssandier

Dpt Système de Référence Temps et Espace

Observatoire de Paris

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# Introduction

The comparison of optical atomic clocks having an accuracy $\delta\nu/\nu \approx 10^{-18}$ onboard a satellite and similar clocks on the ground requires a fully relativistic treatment of light rays within the PPN formalism of metric theories up to the terms of order $1/c^4$.

Indeed for a satellite of altitude $h \sim$ 400 km:

$$\left(\frac{\delta\nu}{\nu}\right)_{Doppler} \approx \frac{v}{c} \sim 2.6\times 10^{-5} \quad \Longrightarrow \quad \frac{v^2}{c^2} \sim 7\times 10^{-10}$$

$$\left(\frac{\delta\nu}{\nu}\right)_{Einstein} \approx \frac{\Delta U}{c^2} \sim 4\times 10^{-11} \quad \quad \frac{U}{c^2} \sim 7\times 10^{-10}\,.$$

Hence the magnitude of the effects of order $1/c^4$ :

$$\frac{U^2}{c^4}, \quad \frac{v^2}{c^2}\frac{U}{c^2}, \quad \frac{v^4}{c^4} \quad \sim \quad 5\times 10^{-19}$$

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## Synchronization. Time transfer functions

For the synchronization in a given coordinate system $x^\alpha = (x^0, \boldsymbol{x})$ we have to know the coordinate travel time $x_B^0 - x_A^0$ of a photon between an emission point $(x_A^0, \boldsymbol{x}_A)$ and a reception point $(x_B^0, \boldsymbol{x}_B)$

- either as a function of $\boldsymbol{x}_A, x_B^0$, and $\boldsymbol{x}_B$:

$$x_B^0 - x_A^0 = c\mathcal{T}_r(\boldsymbol{x}_A, x_B^0, \boldsymbol{x}_B)$$

$\longrightarrow$ $\mathcal{T}_r(\boldsymbol{x}_A, x_B^0, \boldsymbol{x}_B)$ may be called the “reception time transfer function"

- or as a function of $x_A^0, \boldsymbol{x}_A$, and $\boldsymbol{x}_B$:

$$x_B^0 - x_A^0 = c\mathcal{T}_e(x_A^0, \boldsymbol{x}_A, \boldsymbol{x}_B)$$

$\longrightarrow$ $\mathcal{T}_e(x_A^0, \boldsymbol{x}_A, \boldsymbol{x}_B)$ may be called the “emission time transfer function".

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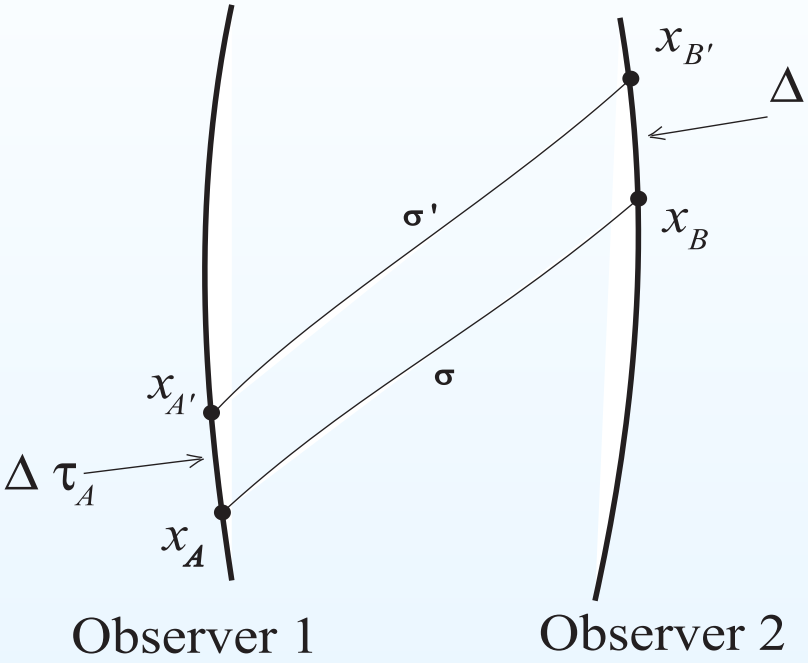
# 1-way frequency transfers




$$\Delta\tau_A = 1/\nu_A$$

$$\Delta\tau_B = 1/\nu_B$$

$$\frac{\nu_A}{\nu_B} = \frac{u_A^0}{u_B^0}\,\frac{1 + c\dfrac{\partial\mathcal{T}_r}{\partial x_A^i}\beta_A^i}{1 - c\dfrac{\partial\mathcal{T}_r}{\partial x_B^0} - c\dfrac{\partial\mathcal{T}_r}{\partial x_B^j}\beta_B^j} = \frac{u_A^0}{u_B^0}\,\frac{1 + c\dfrac{\partial\mathcal{T}_e}{\partial x_A^0} + c\dfrac{\partial\mathcal{T}_e}{\partial x_A^i}\beta_A^i}{1 - c\dfrac{\partial\mathcal{T}_e}{\partial x_B^j}\beta_B^j}, \quad \left(u^0 = \frac{dx^0}{ds},\ \beta^j = \frac{dx^j}{dx^0}\right)$$

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# 3-way/2-way frequency transfers

For an emitter $A$, a transponder $B$ and a receiver $C$:

$$\frac{\nu_C}{\nu_A} = \frac{\nu_C}{\nu_B}\frac{\nu_B}{\nu_A} = \frac{u^0_C}{u^0_A}\,\frac{1 - c\dfrac{\partial \mathcal{T}_e}{\partial x^i_C}\beta^i_C}{1 + c\dfrac{\partial \mathcal{T}_r}{\partial x^i_A}\beta^i_A}\,\frac{1 - c\dfrac{\partial \mathcal{T}_r}{\partial x^0_B} - c\dfrac{\partial \mathcal{T}_r}{\partial x^j_B}\beta^j_B}{1 + c\dfrac{\partial \mathcal{T}_e}{\partial x^0_B} + c\dfrac{\partial \mathcal{T}_e}{\partial x^j_B}\beta^j_B}$$

2-way transfer: the world line of $C$ coincides with the world line of $A$:

- $u^0_A$ and $u^0_C$ taken on the same world line at different times
- $\frac{u^0_C}{u^0_A}$ calculated by a Taylor expansion about the reception (or emision) point : it involves $\boldsymbol{v}_A$, $\boldsymbol{v}_C$, $\boldsymbol{a}_C = d\boldsymbol{v}_C/dt$, $d\boldsymbol{a}_C/dt$ and $d^2\boldsymbol{a}_C/dt^2$ at the order $O(c^{-4})$.

# Parametrized post-Newtonian (PPN) formalism

Using a “geocentric reference system" (GRS) $(x^0 = ct, \boldsymbol{x} = (x^i))$, one assumes that

$$G_{00} = 1 - \frac{2}{c^2}W + \frac{2\beta}{c^4}W^2 + O(5), \qquad G_{ij} = -\left(1 + \frac{2\gamma}{c^2}W\right)\delta_{ij} + O(4),$$

$$G_{0i} = \frac{2}{c^3}(\gamma+1)\,W^i + O(5),$$

$$W(t,\boldsymbol{x}) = W_{(E)}(t,\boldsymbol{x}) + Q_i^{(E)}x^i + W_{(T)}(t,\boldsymbol{x}) + \frac{1}{c^2}\Psi(t,\boldsymbol{x})\,,$$

$$W^i(t,\boldsymbol{x}) = W^i_{(E)}(t,\boldsymbol{x}) + \frac{1}{2}\epsilon_{ijk}C_j(t)x^k + W^i_{(T)}(t,\boldsymbol{x})\,.$$

$W_{(E)}(t,\boldsymbol{x})$ and $W^i_{(E)}(t,\boldsymbol{x})$ are the potential and the gravito-magnetic vector of the Earth
$W_{(T)}(t,\boldsymbol{x})$ and $W^i_{(T)}(t,\boldsymbol{x})$ are tidal potentials of order $\boldsymbol{x}^2$.
$Q_i^{(E)}$ = non geodesic acceleration of the Earth center of mass w.r.t. $(x^0, \boldsymbol{x})$
$\frac{1}{2}\epsilon_{ijk}C_j(t)x^k$ contains Lense-Thirring and de Sitter precessions

$\Psi(t,\boldsymbol{x})$ = possible violation of the Strong Equivalence Principle (SEP).

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# Parametrized post-Newtonian formalism

To determine the time/frequency transfers, we just have to use the following relations:

- Reception time transfer function $\mathcal{T}_r(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B)$:

$$\mathcal{T}_r(\boldsymbol{x}_A, t_B, \boldsymbol{x}_B) = \frac{1}{c}R_{AB} + \frac{1}{c^3}(\gamma+1)R_{AB}\int_0^1 \left[W(z_-^\alpha(\lambda)) - \frac{2}{c}N^i_{AB}.W^i(z_-^\alpha(\lambda))\right] d\lambda + O(5)\,,$$

the integrals being taken along the null straight line in Minkowski space-time defined by

$$z^0_-(\lambda) = -R_{AB}\lambda + ct_B\,, \quad z^i_-(\lambda) = -R_{AB}N^i\lambda + x^i_B\,, \quad 0 \leq \lambda \leq 1\,,$$

$$R_{AB} = |\boldsymbol{x}_B - \boldsymbol{x}_A|\,, \qquad N^i_{AB} = \frac{x^i_B - x^i_A}{R_{AB}}\,.$$

- The time-component $u^0$ for observers $A$ and $B$:

$$\frac{1}{u^0} = \frac{ds}{dx^0} = 1 - \frac{1}{c^2}\left(W + \frac{1}{2}\boldsymbol{v}^2\right)$$
$$+\frac{1}{c^4}\left[\left(\beta - \frac{1}{2}\right)W^2 - \left(\gamma + \frac{1}{2}\right)W\boldsymbol{v}^2 - \frac{1}{8}v^4 + 2(\gamma+1)W^i v^i\right] + O(5)$$

# Frequency shift in the field of the Earth

The frequency shift $\delta\nu/\nu$ is split in two parts:

$$\frac{\delta\nu}{\nu} \equiv \frac{\nu_A}{\nu_B} - 1 = \text{special-relativistic Doppler effect} + \left(\frac{\delta\nu}{\nu}\right)_g ,$$

where $(\delta\nu/\nu)_g$ contains all the contribution of the gravitational field, eventually mixed with kinetic Doppler terms.

Consider now a satellite orbiting around the Earth at $h \approx 400$ km. $A$ is onboard the satellite, $B$ is a ground station.
The Earth is assumed to be axisymmetric about its axis of rotation $Ox^3$.
We get:

$$\left(\frac{\delta\nu}{\nu}\right)_g = \frac{W_A - W_B}{c^2} + \frac{1}{c^3}\left(\frac{\delta\nu}{\nu}\right)^{(3)}_M + \frac{1}{c^3}\left(\frac{\delta\nu}{\nu}\right)^{(3)}_{J_2} + \frac{1}{c^4}\left(\frac{\delta\nu}{\nu}\right)^{(4)}_M + \frac{1}{c^4}\left(\frac{\delta\nu}{\nu}\right)^{(4)}_{G_{0i}} + \cdots$$

where the different terms may be evaluated and discussed as follows.

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# Contributions of $(W_A - W_B)/c^2$

$$W_{(E)}(\boldsymbol{x}) = \frac{GM_{(E)}}{r}\left[1 - \sum_{n=2}^{\infty} J_n \left(\frac{r_e}{r}\right)^n P_n(\cos\theta)\right]$$

- Monopole:

$$\frac{GM_{(E)}}{c^2 r_A} - \frac{GM_{(E)}}{c^2 r_B} \approx -4 \times 10^{-11}$$

- Mass multipoles:

$$J_2 = 1.0826 \times 10^{-3},$$
$$J_3 = 2.53 \times 10^{-6},\ J_4 = 1.62 \times 10^{-6},$$
$$J_5 = 2.28 \times 10^{-7},\ J_6 = 5.41 \times 10^{-7}, \ldots$$

From NASA data for ISS (C. Le Poncin-Lafitte & S Lambert, 2007):

$$\left|\frac{1}{c^2}\left(\frac{\delta\nu}{\nu}\right)_{J_2}\right| \leq 2 \times 10^{-13}, \qquad \left|\frac{1}{c^2}\left(\frac{\delta\nu}{\nu}\right)_{J_n}\right| \leq 10^{-16} \quad \text{for } n = 3, 4, 5, 6$$

But problems with convergence on the ground level.

# Contributions of $(W_A - W_B)/c^2$

- Tidal potentials due to the external bodies $K$ (Sun, Moon, Jupiter,...):

$$W_{(T)}(t,\boldsymbol{x}) = \sum_{K\neq E} \frac{GM_{(K)}}{d_{(K)}} \sum_{n=2}^{\infty} P_n\left(\frac{\boldsymbol{d}_{(K)}}{d_{(K)}}.\frac{\boldsymbol{x}}{r}\right)\left(\frac{r}{d_{(K)}}\right)^n$$

where $\boldsymbol{d}_{(K)}$ is the vector position of the center of $K$ and $r = |\boldsymbol{x}|$.

$$\left|W_{(T,K)}(t,\boldsymbol{x})\right| \leq 3 \times 10^{-17}$$

The tidal terms must be taken into account.

- Contribution of $Q_i x^i$:

$$Q_i = -\frac{1}{2M_{(E)}} I^{km}_{(E)} \partial^3_{ikm}\left(\sum_{K\neq E} U_{(K)}(\boldsymbol{x}_E)\right) + \ldots \quad \Longrightarrow \quad \left|\frac{1}{c^2} Q^i x^i\right| \leq 10^{-20}$$

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# Contributions of $(W_A - W_B)/c^2$

- Contribution of $\Psi(t, \boldsymbol{x})$:

$$\Psi(t,\boldsymbol{x}) \approx -(4\beta-\gamma-1)\left[W_{(E)}\left(\sum_{K\neq E} U_{(K)}(\boldsymbol{x}_E) - \frac{GM_\odot}{D^3}\boldsymbol{D}.\boldsymbol{x}\right) + \frac{1}{2}\frac{GM_{(E)}}{r}\frac{GM_\odot}{D^3}\boldsymbol{D}.\boldsymbol{x}\right]$$

where $\eta = 4\beta - \gamma - 1)$ is the Nordvedt parameter indicating a violation of the strong equivalence principle, $D$ is the distance Sun-Earth, $M_\odot$ the mass of the Sun.

The Sun is the dominant external body in $\sum_{K\neq E} U_{(K)}(\boldsymbol{x}_E)$. So

$$|\eta| \leq \times 10^{-3} \quad \Longrightarrow \quad |\Psi(t,\boldsymbol{x})| \leq 10^{-21}$$

A violation of the SEP cannot be observed with this configuration.

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# Influence of the mass monopole at order $O(3)$

$$\left(\frac{\delta\nu}{\nu}\right)^{(3)}_M = -\frac{GM(r_A+r_B)}{r_A r_B}\left[\left(\frac{\gamma+1}{1+\boldsymbol{n}_A\cdot\boldsymbol{n}_B} - \frac{r_A-r_B}{r_A+r_B}\right)\boldsymbol{N}_{AB}\cdot(\boldsymbol{v}_A-\boldsymbol{v}_B) + (\gamma+1)\frac{R_{AB}}{r_A+r_B}\frac{\boldsymbol{n}_A\cdot\boldsymbol{v}_A+\boldsymbol{n}_B\cdot\boldsymbol{v}_B}{1+\boldsymbol{n}_A\cdot\boldsymbol{n}_B}\right],$$

where

$$\boldsymbol{n}_A = \frac{\boldsymbol{x}_A}{r_A}, \quad \boldsymbol{n}_B = \frac{\boldsymbol{x}_B}{r_B}, \quad \boldsymbol{N}_{AB} = \frac{\boldsymbol{x}_B-\boldsymbol{x}_A}{R_{AB}}.$$

So

$$\left(\frac{\delta\nu}{\nu}\right)^{(3)}_M \sim 5\times 10^{-14}$$

# Influence of the quadrupole moment $J_2$ at order $O(3)$

With $\boldsymbol{N}_{AB} = (\boldsymbol{x}_B - \boldsymbol{x}_A)/R_{AB}\,,\ K_{AB} = (r_A - r_B)^2/r_A r_B$ :

$$\begin{aligned}
\left(\frac{\delta\nu}{\nu}\right)^{(3)}_{J_2} \quad = \quad & \frac{GM}{2r_e} J_2 \left(\boldsymbol{N}_{AB}\cdot(\boldsymbol{v}_A - \boldsymbol{v}_B)\right) \left[\left(\frac{r_e}{r_A}\right)^3 \left[3(\boldsymbol{k}\cdot\boldsymbol{n}_A)^2 - 1\right]\right. \\
& \left. - \left(\frac{r_e}{r_B}\right)^3 \left[3(\boldsymbol{k}\cdot\boldsymbol{n}_B)^2 - 1\right]\right] \\
& + \frac{\gamma+1}{2}\,\frac{GM\,J_2 r_e^2 (r_A + r_B)}{r_A^2 r_B^2}\,\frac{1}{(1+\boldsymbol{n}_A\cdot\boldsymbol{n}_B)^2} \\
& \times \left\{ (\boldsymbol{N}_{AB}\cdot(\boldsymbol{v}_A - \boldsymbol{v}_B)) \left[(\boldsymbol{k}\cdot\boldsymbol{n}_A + \boldsymbol{k}\cdot\boldsymbol{n}_B)^2\,\frac{5 - 3\boldsymbol{n}_A\cdot\boldsymbol{n}_B + 2K_{AB}}{1+\boldsymbol{n}_A\cdot\boldsymbol{n}_B}\right.\right. \\
& \left. - \left(1 - \frac{r_A(\boldsymbol{k}\cdot\boldsymbol{n}_B)^2 + r_B(\boldsymbol{k}\cdot\boldsymbol{n}_A)^2}{r_A + r_B}\right)(3 - \boldsymbol{n}_A\cdot\boldsymbol{n}_B + K_{AB})\right] \\
& + \frac{R_{AB}}{r_A + r_B}(\boldsymbol{n}_A\cdot\boldsymbol{v}_A + \boldsymbol{n}_B\cdot\boldsymbol{v}_B)(\boldsymbol{k}\cdot\boldsymbol{n}_A + \boldsymbol{k}\cdot\boldsymbol{n}_B)^2\,\frac{7 - \boldsymbol{n}_A\cdot\boldsymbol{n}_B + 2K_{AB}}{1+\boldsymbol{n}_A\cdot\boldsymbol{n}_B}
\end{aligned}$$

# Influence of the quadrupole moment $J_2$ at order $O(3)$ (continued)

$$
\begin{aligned}
&-\frac{R_{AB}}{r_A}(\boldsymbol{n}_A\cdot\boldsymbol{v}_A)\left[1-3(\boldsymbol{k}\cdot\boldsymbol{n}_A)^2\right]\frac{r_A+r_B(2+\boldsymbol{n}_A\cdot\boldsymbol{n}_B)}{r_A+r_B}\\
&-\frac{R_{AB}}{r_B}(\boldsymbol{n}_B\cdot\boldsymbol{v}_B)\left[1-3(\boldsymbol{k}\cdot\boldsymbol{n}_B)^2\right]\frac{r_A(2+\boldsymbol{n}_A\cdot\boldsymbol{n}_B)+r_B}{r_A+r_B}\\
&+R_{AB}\Bigg[2\left(\frac{\boldsymbol{n}_A\cdot\boldsymbol{v}_A}{r_A}+\frac{\boldsymbol{n}_B\cdot\boldsymbol{v}_B}{r_B}\right)(\boldsymbol{k}\cdot\boldsymbol{n}_A)(\boldsymbol{k}\cdot\boldsymbol{n}_B)\\
&\qquad-(\boldsymbol{n}_A\cdot\boldsymbol{v}_A)\frac{1-(\boldsymbol{k}\cdot\boldsymbol{n}_B)^2}{r_B}-(\boldsymbol{n}_B\cdot\boldsymbol{v}_B)\frac{1-(\boldsymbol{k}\cdot\boldsymbol{n}_A)^2}{r_A}\Bigg]\\
&-2\,\frac{R_{AB}}{r_A}(\boldsymbol{k}\cdot\boldsymbol{v}_A)\left[\boldsymbol{k}\cdot\boldsymbol{n}_A\frac{r_A+r_B(2+\boldsymbol{n}_A\cdot\boldsymbol{n}_B)}{r_A+r_B}+\boldsymbol{k}\cdot\boldsymbol{n}_B\right]\\
&-2\,\frac{R_{AB}}{r_B}(\boldsymbol{k}\cdot\boldsymbol{v}_B)\left[\boldsymbol{k}\cdot\boldsymbol{n}_A+\boldsymbol{k}\cdot\boldsymbol{n}_B\frac{r_A(2+\boldsymbol{n}_A\cdot\boldsymbol{n}_B)+r_B}{r_A+r_B}\right]\Bigg\}.
\end{aligned}
$$

Crude estimate:

$$
\left|\frac{1}{c^3}\left(\frac{\delta\nu}{\nu}\right)^{(3)}_{J_2}\right|\leq 1.3\times 10^{-16}.
$$

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# Influence of the mass monopole at order $O(4)$

$$\left(\frac{\delta\nu}{\nu}\right)^{(4)}_M = (\gamma+1)\left(\frac{GM}{r_A}v_A^2 - \frac{GM}{r_B}v_B^2\right) - \frac{GM(r_A-r_B)}{2\,r_A r_B}(v_A^2 - v_B^2)$$
$$+\frac{1}{2}\left(\frac{GM}{r_A r_B}\right)^2\left[(r_A-r_B)^2 + 2(\beta-1)(r_A^2 - r_B^2)\right]$$
$$-\frac{GM(r_A+r_B)}{r_A r_B}\left[\left(\frac{2(\gamma+1)}{1+\boldsymbol{n}_A\cdot\boldsymbol{n}_B} - \frac{r_A-r_B}{r_A+r_B}\right)(\boldsymbol{N}_{AB}\cdot(\boldsymbol{v}_A-\boldsymbol{v}_B))\,(\boldsymbol{N}_{AB}\cdot\boldsymbol{v}_B)\right.$$
$$\left.+\frac{\gamma+1}{1+\boldsymbol{n}_A\cdot\boldsymbol{n}_B}\frac{R_{AB}}{r_A+r_B}\left((\boldsymbol{n}_A\cdot\boldsymbol{v}_A)\,(\boldsymbol{N}_{AB}\cdot\boldsymbol{v}_B) - (\boldsymbol{N}_{AB}\cdot(\boldsymbol{v}_A-2\boldsymbol{v}_B))(\boldsymbol{n}_B\cdot\boldsymbol{v}_B)\right)\right].$$

The dominant term:

$$(\gamma+1)GMv_A^2/r_A \;\sim\; 10^{-18}$$

It will probably be necessary to take this correction into account in experiments performed with optical clocks.

## Influence of the $G_{0i}$ potentials

- Contribution of the de Lense-Thirring precession due to the Earth:

$$\begin{aligned}\left(\frac{\delta\nu}{\nu}\right)^{(4)}_{\boldsymbol{S}} = & (\gamma+1)\frac{GS_{(E)}}{r_A^2}\left(1+\frac{r_A}{r_B}\right)\boldsymbol{v}_A\cdot\left\{\frac{\boldsymbol{k}\times\boldsymbol{n}_B}{1+\boldsymbol{n}_A\cdot\boldsymbol{n}_B}-\frac{r_B}{r_A+r_B}\boldsymbol{k}\times\boldsymbol{n}_A\right.\\ & \left.+\frac{\boldsymbol{k}\cdot(\boldsymbol{n}_A\times\boldsymbol{n}_B)}{(1+\boldsymbol{n}_A\cdot\boldsymbol{n}_B)^2}\left[\frac{r_A+r_B(2+\boldsymbol{n}_A\cdot\boldsymbol{n}_B)}{r_A+r_B}\boldsymbol{n}_A+\boldsymbol{n}_B\right]\right\},\\ & -(\gamma+1)\frac{GS}{r_B^2}\left(1+\frac{r_B}{r_A}\right)\boldsymbol{v}_B\cdot\left\{\frac{\boldsymbol{k}\times\boldsymbol{n}_A}{1+\boldsymbol{n}_A\cdot\boldsymbol{n}_B}-\frac{r_A}{r_A+r_B}\boldsymbol{k}\times\boldsymbol{n}_B\right.\\ & \left.-\frac{\boldsymbol{k}\cdot(\boldsymbol{n}_A\times\boldsymbol{n}_B)}{(1+\boldsymbol{n}_A\cdot\boldsymbol{n}_B)^2}\left[\boldsymbol{n}_A+\frac{r_A(2+\boldsymbol{n}_A\cdot\boldsymbol{n}_B)+r_B}{r_A+r_B}\boldsymbol{n}_B\right]\right\}.\end{aligned}$$

$S_{(E)}$ = intrinsic angular momentum of the Earth. Crude estimate:

$$\left|\frac{1}{c^4}\left(\frac{\delta\nu}{\nu}\right)^{(4)}_{\boldsymbol{S}}\right| \leq (\gamma+1)\times 10^{-19}.$$

- Contribution of the de Sitter precession due to the Sun: $\left|\frac{1}{c^4}\left(\frac{\delta\nu}{\nu}\right)^{(4)}_{\boldsymbol{S}}\right| \sim 10^{-22}$.

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