Abstract
We have developed a fiber coupled cryogenic optical sapphire resonator for the frequency stabilization of a Nd:YAG laser. With this method a frequency instability of \(2 \cdot 10^{-15}\) for \(\tau = 1\) hour integration time has been achieved. This is among the lowest values ever obtained for optical oscillators. We also present a new approach to test the time-independence of the fine structure constant, based on the use of a single monolithic resonator. Preliminary results on the development of suitable resonators are described.

Introduction
The development of cryogenic optical resonators (COREs) is motivated by the need for more stable reference resonators for high-resolution spectroscopy, and as a tool for fundamental physics tests. With sufficient long-term stability (integration times of months), resonators could play a role in performing improved tests of special and general relativity.

In previous work we had shown that with cryogenic sapphire resonators an instability of \(3 \cdot 10^{-15}\) can be reached at \(\tau = 1\) min [1]. Moreover, the resonators do not exhibit any discernible drift, with the current limit being less than 4 kHz over 6 months [2]. In those experiments the laser beams were coupled to the COREs through windows in the cryostat, and on time scales longer than tens of minutes the drift of the laser beam alignment limited the stability of the lock. We have therefore proceeded to implement a coupling via a fiber whose end is stably positioned relative to the CORE.

Cryogenic fiber-coupled sapphire reference cavities
The basic set-up we have employed is shown in Fig. 1. All components, especially fiber end and CORE, are maintained in relative positions that are very stable, thanks to the small thermal expansion of components and housing at liquid helium temperature. The monomode nonpolarizing fiber is glued into a holder that is attached to the aluminum housing. The light is mode-matched to the CORE by a micro lens (L). A polarizer (P) ensures that the polarization interrogating the CORE is fixed. A beam splitter (BS) serves to detect the light reflected from the cavity and to obtain an error signal for frequency locking. Photodiodes (PD) are also provided to monitor the incident light power and the power transmitted through the cavity, in order to be able to study systematic effects. The photodiodes are standard InGaAs, with shielded cables leading to the amplifiers mounted on the liquid nitrogen heat shield. The fiber coupling was implemented on the CORE that in previous CORE-CORE comparisons had shown the largest sensitivity to laser beam alignment changes. We then performed a beat frequency measurement between the fiber-coupled CORE and another, free-space-coupled CORE located in a different cryostat on the same optical table.

Figure 1: Schematic of the fiber-coupled CORE.

Figure 2: 100 h beat between a fiber-coupled and a free beam coupled CORE, and the resulting Allan deviation.

The reasons responsible for the occurrence of the floor are currently under investigation.
A new approach for a test of $da/dt$

Here we propose a method for a test of the timeindependence of $\alpha$ necessitating only a single resonator, which must contain a dispersive medium. A potentially feasible implementation could be based on a monolithic crystalline resonator. Assuming the bound electron model of the polarizability, with a resonance frequency that scales with the Rydberg energy and thus with $\alpha^2$, the index of refraction $n$ of a medium has the following dependence on $\alpha$: \[ \frac{dn(\omega)}{d\alpha} = -\frac{2}{n} \frac{dn(\omega)}{d\omega}. \] Two approaches are possible. In the first (Fig. 3), two frequencies $\omega_1$, $\omega_2$ are locked to an optically isotropic monolithic resonator. Their frequency difference is measured as a function of time. If $\alpha$ is time-dependent, a change in the difference $\omega_1 - \omega_2$ results from the different dispersion \[ \frac{dn(\omega_1)}{d\omega} \neq \frac{dn(\omega_2)}{d\omega}. \] In practice, $\omega_2$ should be close to a harmonic of $\omega_1$ so that a single laser $\omega_1$ can produce $\omega_2$ using standard nonlinear optical frequency conversion.

Alternatively, a dispersive anisotropic medium can be used. One can then lock two orthogonally polarized waves (o and e) of nearly the same frequency to the birefringent cavity, making use of the fact that their dispersion differs, \[ \frac{dn_{\omega_1, e}(\omega_1)}{d\omega} \neq \frac{dn_{\omega_1, o}(\omega_2)}{d\omega}. \] In this case $\omega_2$ is generated from $\omega_1$ by acoustooptic or electrooptic modulation. A combination of both effects could also be used.

An estimate of the effect can easily be derived. Taking $\omega_2 = m \cdot \omega_f$ (for practical reasons, $m$ will be an integer) and $\omega_1 \approx \omega_f$, the change in the beat frequency due to a supposed change $\Delta \alpha$ may be written as: \[ \Delta(\omega_1 - \omega_f) = A(\Delta \alpha/\alpha + \Delta L/L) + \Delta \Omega_1 - \Delta \Omega_2/m + \omega_1(1 + n\bar{\omega}^2)^{-1}\Delta n_2/n_2 - (1 + n\bar{\omega})^{-1}\Delta n_1/n_1. \] This expression includes the $\alpha$-dependence of the characteristic lifetime $L \sim \alpha^{-1}$. The influence of drifts of the resonator length, $\Delta L$, and the indices, $\Delta n$, due to creep or temperature drift, as well as of lock errors, $\Delta \Omega$, has also been included. Here $A = \omega_1(1 + n\bar{\omega})^{-1} - (1 + n\bar{\omega})^{-1}$ is a material parameter, with the normalized dispersion $n = n\bar{\omega}dn/d\omega$. In the case of a birefringent cavity, $n_1 = n_{\omega_1, e}(\omega_1)$, $n_2 = n_{\omega_1, o}(\omega_2)$. The table shows values for $A = |A| \cdot 10^{-12}$ Hz/2$\pi$ for a few low-loss materials, taking $\lambda_1 = 1064$ nm and $\lambda_2 = 532$ nm.

<table>
<thead>
<tr>
<th>Material</th>
<th>$A^{1,\lambda_1}$</th>
<th>$A^{1,\lambda_2}$</th>
<th>$A^{1,\lambda_1}$</th>
<th>$A^{1,\lambda_2}$</th>
<th>$A^{1,\lambda_1}$</th>
<th>$A^{1,\lambda_2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CaF$_2$</td>
<td>2.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>MgF$_2$</td>
<td>1.3</td>
<td>1.4</td>
<td>1.3</td>
<td>1.3</td>
<td>0.60</td>
<td>0.2</td>
</tr>
<tr>
<td>Al$_2$O$_3$</td>
<td>4.7</td>
<td>4.6</td>
<td>4.7</td>
<td>4.6</td>
<td>0.03</td>
<td>0.2</td>
</tr>
<tr>
<td>SiO$_2$</td>
<td>3.0</td>
<td>3.2</td>
<td>2.9</td>
<td>3.1</td>
<td>0.10</td>
<td>0.3</td>
</tr>
</tbody>
</table>

We can deduce that in order to achieve a limit $|A^{-1}da/dt| < 2 \cdot 10^{-15}/$year in an integration time of 1 month, a beat frequency resolution of 0.1 mHz must be available, implying that over this time the locking errors $\Delta \Omega/2\pi$ must be below 0.1 mHz, the relative drifts below $1 \cdot 10^{-16}$ and the refractive index drift below $1 \cdot 10^{-18}$, if $\Delta \Omega_1$ and $\Delta \Omega_2$ are uncorrelated. These values are extremely low and one will necessarily have to use a cryogenic resonator with very narrow linewidth.

**Experimental results**

We have studied the properties of a 4 cm long birefringent monolithic standing-wave resonator made out of nominally highly pure single-crystal sapphire, with dielectric mirrors for 1064 nm coated on the endfaces. The crystal axis was oriented perpendicular to the cavity axis. We have measured mirror transmissions of less than 2 ppm, but the internal losses were high, 600 ppm per round-trip, leading to strong impedance-mismatch, and very low incoupling. A laser calorimetric measurement of the crystal absorption gave 20 ppm/cm (V. Lorriette and A.C. Boccazar, ESPCI Paris), indicating that additional loss was present in the resonator.

Due to the difficulty in finding a source of ultrapure sapphire that could lead to a suitable monolithic resonator, single-crystals of CaF$_2$ and MgF$_2$ were studied as possible alternatives. Calorimetry showed extremely low loss at 1064 nm, below 2 ppm/cm. These materials are thus promising for the next generation of crystalline monolithic resonators, and their fabrication and testing is under way.

**Conclusion**

A significant improvement of CORE stability for $\tau = 1$ hour has been demonstrated. We believe further improvements will be possible by enhancing the signal-to-noise ratio of the error signal. The possibility of performing a test of $da/dt$ with a single resonator is a strong motivation to develop monolithic COREs. This requires a search for a crystalline material with unprecedented low optical loss. Potential candidates have been identified.

**REFERENCES**